Lecture: Corporate and Personal Income Tax

Lutz Kruschwitz & Andreas Löffler

Discounted Cash Flow, Section 5

Remark: The slightly expanded second edition (Springer, open access) has different enumeration than the first (Wiley). We use Springer's enumeration in the slides and Wiley's in the videos.

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Outline

Again we have an unlevered firm (self-financed, distributing its cash flows fully) and a levered firm (indebted, partial retention of cash flows). The levered firm lives for ever. For simplicity we will assume for the levered firm

- that debt D remains constant, and
- that a constant amount A is retained every period.

Corporate and personal income tax 2

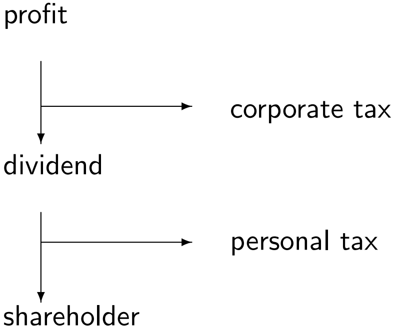
We consider a corporate and a personal income tax.

The **corporate tax** is measured by the company's profit. The tax rate is τ^C and independent of time. Taking out loans at time $t-1$ provides a tax advantage equal to $\tau^C \widetilde{I}_t$.

The **personal tax** is measured by the paid dividend (tax rate τ^D) and the paid interest as well (tax rate τ^I). Again, the tax is linear and independent of time. If $\widetilde{A}_t$ is retained this amount creates a tax advantage.

Interaction of both taxes 3

Consider a shareholder of a company that tries to distribute its profit. The tax authorities might have access to the profit twice:



Two ways to deal with this problem

- Profits are taxed twice (double taxation or classical system).
- The tax authority can accept that the corporate income tax is considered as a first installment (indirect relief or imputation system).
- *Both systems can be mixed.*

We will consider a classical system from now on. Several papers deal with other models of taxation.

From pre-tax gross cash flows to post-tax free cash flows 5

	Gross cash flow before taxes	$\widetilde{GCF}_t$
$-$	Corporate income taxes	$\widetilde{Tax}_t^C$
$-$	Investment expenses	$\widetilde{Inv}_t$
$-$	Interest (creditor's taxable income)	$\widetilde{I}_t$
$-$	Debt repayments	$-\widetilde{D}_t + \widetilde{D}_{t-1}$
$-$	Retained earnings	$\widetilde{A}_t$
$+$	Reflux from retained earnings	$(1 + \widetilde{r}_{t-1})\widetilde{A}_{t-1}$
$-$	Shareholder's personal income tax	$\widetilde{Tax}_t^P$
$=$	Shareholder's levered post-tax cash flow	$\widetilde{FCF}_t^1$

For the levered as well as the unlevered firm

$$\widetilde{FCF}_t = \widetilde{GCF}_t - \widetilde{Tax}_t^C - \widetilde{Inv}_t - \widetilde{Tax}_t^P \,.$$

That implies for the tax shield,

$$\widetilde{FCF}_t^{\mathrm{l}} = \widetilde{FCF}_t^{\mathrm{u}} - \widetilde{I}_t + \widetilde{D}_t - \widetilde{D}_{t-1} - \widetilde{A}_t + (1 + \widetilde{r}_{t-1})\widetilde{A}_{t-1} + \widetilde{Tax}_t^{C,u} - \widetilde{Tax}_t^{C,l} + \widetilde{Tax}_t^{P,u} - \widetilde{Tax}_t^{P,l} \,.$$

Let us now turn to the tax base of the corporate as well as the personal income tax.

For the corporate income tax

$$\begin{aligned}\widetilde{Tax}_t^{C,l} &= \tau^C \widetilde{EBT}_t^{\mathrm{l}} \\ &= \tau^C \left(\widetilde{EBT}_t^{u} - r_f D + \widetilde{r}_{t-1} A \right) \\ &= \widetilde{Tax}_t^{C,u} - \tau^C r_f D + \tau^C \widetilde{r}_{t-1} A\end{aligned}$$

and for the personal income tax

$$\begin{aligned}\widetilde{Tax}_t^{P,l} &= \widetilde{Tax}_t^{P,u} - \tau^I r_f D + \tau^D \widetilde{r}_{t-1} A + \tau^I \tau^C r_f D - \tau^D \tau^C \widetilde{r}_{t-1} A \\ &= \widetilde{Tax}_t^{P,u} - \tau^I \left(1 - \tau^C\right) r_f D + \tau^D \left(1 - \tau^C\right) \widetilde{r}_{t-1} A \,.\end{aligned}$$

We have

$$\widetilde{FCF}^{\mathrm{l}}_t = \widetilde{FCF}^{\mathrm{u}}_t - r_f D + \widetilde{r}_{t-1} A + \widetilde{Tax}^{C,u} - \widetilde{Tax}^{C,l} + \widetilde{Tax}^{P,u} - \widetilde{Tax}^{P,l}$$
$$= \widetilde{FCF}^{\mathrm{u}}_t - \left(1-\tau^I\right)\left(1-\tau^C\right) r_f D + \left(1-\tau^D\right)\left(1-\tau^C\right) \widetilde{r}_{t-1} A$$

which gives

$$\mathrm{E}_Q\left[\widetilde{FCF}^{\mathrm{l}}_t | \mathcal{F}_{t-1}\right] = \mathrm{E}_Q\left[\widetilde{FCF}^{\mathrm{u}}_t | \mathcal{F}_{t-1}\right] + \left(1-\tau^D\right)\left(1-\tau^C\right) r_f A - \left(1-\tau^I\right)\left(1-\tau^C\right) r_f D\ .$$

Using our fundamental theorem we get

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\begin{aligned}
\widetilde{V}_t^{\mathrm{l}} &= \widetilde{V}_t^{\mathrm{u}} + D + \sum_{s=t+1}^{\infty} \frac{\mathsf{E}_Q\left[\left(1-\tau^D\right)\left(1-\tau^C\right) r_f A - \left(1-\tau^I\right)\left(1-\tau^C\right) r_f D | \mathcal{F}_t\right]}{\left(1+r_f\left(1-\tau^I\right)\right)^{s-t}} \\
&= \widetilde{V}_t^{\mathrm{u}} + D + \sum_{s=t+1}^{\infty} \frac{\left(1-\tau^D\right)\left(1-\tau^C\right)}{\left(1+r_f\left(1-\tau^I\right)\right)^{s-t}} r_f A - \sum_{s=t+1}^{\infty} \frac{\left(1-\tau^I\right)\left(1-\tau^C\right)}{\left(1+r_f\left(1-\tau^I\right)\right)^{s-t}} r_f D \\
&= \widetilde{V}_t^{\mathrm{u}} + D + \frac{\left(1-\tau^D\right)\left(1-\tau^C\right)}{r_f\left(1-\tau^I\right)} r_f A - \frac{\left(1-\tau^I\right)\left(1-\tau^C\right)}{r_f\left(1-\tau^I\right)} r_f D \\
&= \widetilde{V}_t^{\mathrm{u}} + \frac{\left(1-\tau^D\right)\left(1-\tau^C\right)}{1-\tau^I} A + \tau^C D \,,
\end{aligned}
$$

which is a generalization of Modigliani-Miller.