Lecture: Retention Policies

Lutz Kruschwitz & Andreas Löffler

Discounted Cash Flow, Section 4.3

Remark: The slightly expanded second edition (Springer, open access) has different enumeration than the first (Wiley). We use Springer's enumeration in the slides and Wiley's in the videos.

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Outline

Retention policies 1

We analyse four different retention policies:

- autonomous retention,
- retention based on cash flows,
- retention based on dividends,
- retention based on market values.

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Autonomous retention 2

Definition: *A firm follows autonomous retention if $\widetilde{A}_t$ is deterministic.*

Valuation: *The following equations holds*

$$\widetilde{V}_t^{\mathrm{l}} = \widetilde{V}_t^{\mathrm{u}} + \left(1-\tau^D\right) A_t + \frac{\tau^I \left(1-\tau^D\right) r_f A_t}{1+r_f\left(1-\tau^I\right)} + \ldots + \frac{\tau^I \left(1-\tau^D\right) r_f A_{T-1}}{\left(1+r_f\left(1-\tau^I\right)\right)^{T-t}}.$$

If, in particular, retention is constant and the firm lives on for ever, then

$$\widetilde{V}_t^{\mathrm{l}} = \widetilde{V}_t^{\mathrm{u}} + \frac{1-\tau^D}{1-\tau^I} A.$$

Let us assume that

$$A_0 = 10, \quad A_1 = 20, \quad A_2 = 0\,.$$

With that we get

$$\begin{aligned} V_0^{\mathrm{l}} &= V_0^{\mathrm{u}} + \widetilde{V}_t^{\mathrm{u}} + (1-\tau)A_0 + \frac{\tau(1-\tau)r_f A_0}{1+r_f(1-\tau)} + \frac{\tau(1-\tau)r_f A_1}{(1+r_f(1-\tau))^2} \\ &= 249.692 + (1-0.5)\times 10 + \frac{0.5\times(1-0.5)\times 0.1\times 10}{1+0.1\times(1-0.5)} \\ &\quad + \frac{0.5\times(1-0.5)\times 0.1\times 20}{(1+0.1\times(1-0.5))^2} \approx 255.383 \end{aligned}$$

for the value of the levered firm.

Definition: *Retention is based on cash flows if the firm retains a determined percentage of cash flows*

$$\widetilde{A}_t = \alpha_t \, \widetilde{FCF}^{\mathrm{u}}_t \,.$$

Valuation: *The following equations holds*

$$\widetilde{V}^{\mathrm{l}}_t = \widetilde{V}^{\mathrm{u}}_t + \frac{(1+r_f)\left(1-\tau^D\right)\alpha_t \widetilde{FCF}^{\mathrm{u}}_t}{1+r_f\left(1-\tau^I\right)}$$
$$+\frac{\tau^I r_f\left(1-\tau^D\right)}{1+r_f\left(1-\tau^I\right)}\left(\frac{\mathrm{E}\left[\alpha_{t+1}\widetilde{FCF}^{\mathrm{u}}_{t+1}|\mathcal{F}_t\right]}{1+k^{E,\mathrm{u}}}+\ldots+\frac{\mathrm{E}\left[\alpha_{T-1}\widetilde{FCF}^{\mathrm{u}}_{T-1}|\mathcal{F}_t\right]}{(1+k^{E,\mathrm{u}})^{T-t}}\right).$$

Proof 5

Quite straightforward:

$$\begin{aligned}\widetilde{V}^{\mathrm{l}}_t &= \widetilde{V}^{\mathrm{u}}_t + \left(1-\tau^D\right)\widetilde{A}_t + \frac{\mathsf{E}_Q\left[\tau^I r_f \left(1-\tau^D\right)\widetilde{A}_t|\mathcal{F}_t\right]}{1+r_f\left(1-\tau^I\right)} + \ldots + \frac{\mathsf{E}_Q\left[\tau^I r_f\left(1-\tau^D\right)\widetilde{A}_{T-1}|\mathcal{F}_t\right]}{\left(1+r_f\left(1-\tau^I\right)\right)^{T-t}} \\ &= \widetilde{V}^{\mathrm{u}}_t + \left(1-\tau^D\right)\alpha_t \widetilde{FCF}^{\mathrm{u}}_t + \frac{\tau^I r_f\left(1-\tau^D\right)\alpha_t\widetilde{FCF}^{\mathrm{u}}_t}{1+r_f\left(1-\tau^I\right)} \\ &\quad + \frac{\tau^I r_f\left(1-\tau^D\right)}{1+r_f\left(1-\tau^I\right)}\left(\frac{\mathsf{E}_Q\left[\alpha_{t+1}\widetilde{FCF}^{\mathrm{u}}_{t+1}|\mathcal{F}_t\right]}{1+r_f\left(1-\tau^I\right)} + \ldots + \frac{\mathsf{E}_Q\left[\alpha_{T-1}\widetilde{FCF}^{\mathrm{u}}_{T-1}|\mathcal{F}_t\right]}{\left(1+r_f\left(1-\tau^I\right)\right)^{T-t}}\right)\end{aligned}$$

and use Theorem 4.4.

If the firm lives for ever and α is constant, we get

$$\widetilde{V}_t^{\mathrm{l}} = \left(1 + \frac{\tau^I r_f \left(1 - \tau^D\right) \alpha}{1 + r_f \left(1 - \tau^I\right)}\right) \widetilde{V}_t^{\mathrm{u}} + \frac{\left(1 + r_f\right) \left(1 - \tau^D\right) \alpha}{1 + r_f \left(1 - \tau^I\right)} \widetilde{FCF}_t^{\mathrm{u}} .$$

We assume

$$\alpha_0 = 0\,\%, \quad \alpha_1 = 10\,\%\,, \quad \alpha_2 = 20\,\%\,.$$

With that we get

$$\begin{aligned} V_0^{\mathrm{l}} &= V_0^u + \frac{(1+r_f)(1-\tau)\alpha_0 FCF_0^u}{1+r_f(1-\tau)} + \frac{\tau r_f(1-\tau)}{1+r_f(1-\tau)} \left(\alpha_1 \frac{\operatorname{E}\left[\widetilde{FCF}_1^{\mathrm{u}}\right]}{1+k^{E,\mathrm{u}}} \right. \\ &\qquad \left. + \alpha_2 \frac{\operatorname{E}\left[\widetilde{FCF}_2^{\mathrm{u}}\right]}{(1+k^{E,\mathrm{u}})^2} \right) \\ &= 249.692 + 0 + \frac{0.5\times 0.1\times(1-0.5)}{1+0.1\times(1-0.5)} \times \left(0.1 \times \frac{100}{1+0.15} + 0.2 \times \frac{110}{(1+0.15)^2} \right) \\ &\approx 250.295 \end{aligned}$$

for the value of the levered firm.

Retention is based on dividends if until $n \ll T$ the dividend is determined. We furthermore assume that the investment policy is given and that the free cash flows suffice to pay this dividend. Hence, the retention has to be

$$\widetilde{A}_t = \left(\frac{1}{1-\tau^D}\widetilde{FCF}^{\mathrm{u}}_t + (1+\widetilde{r}_{t-1})\widetilde{A}_{t-1} - \ Div_t\right)^+$$

for all $t = 1, \ldots, n$.
Because of the maximum function **without any further assumptions** a valuation equation has to take **options into account**. If we assume that the **retention stays positive** or

$$\frac{1}{1-\tau^D}\widetilde{FCF}^{\mathrm{u}}_t \geq Div_t \qquad \forall t \leq n$$

then a closed form solution will be possible.

Valuation: *The valuation equation then reads*

$$\widetilde{V}_u^{\mathrm{l}} = \widetilde{V}_t^{\mathrm{u}} + \tau^I\left(1-\tau^D\right)\left(\frac{1+r_f}{1+r_f\left(1-\tau^I\right)}\right)^{n-t+1}\widetilde{A}_t$$
$$+\tau^I r_f \sum_{v=t+1}^{n}\left(\frac{\mathrm{E}\left[\widetilde{FCF}_v^{\mathrm{u}}|\mathcal{F}_t\right]}{(1+k^{E,\mathrm{u}})^{v-t}} - \frac{\left(1-\tau^D\right)Div_v}{\left(1+r_f\left(1-\tau^I\right)\right)^{v-t}}\right)\left(1+\left(\frac{1+r_f}{1+r_f\left(1-\tau^I\right)}\right)^{n+1-v}\right).$$

We consider generalization for $n \to \infty$ as being useless (then the formula might violate the transversality condition).

Let us assume

$$A_0 = 0, \quad Div_1 = Div_2 = 40.$$

If the tax rate amounts to 50 %, cash flows are greater than the dividend. Under these conditions we get

$$V_0^{\mathrm{l}} = V_0^u + \tau r_f \left(\frac{\mathrm{E}\left[\widetilde{FCF}_1^{\mathrm{u}}\right]}{1+k^{E,\mathrm{u}}} - \frac{(1-\tau)Div_1}{1+(1-\tau)r_f} \right) \left(1 + \left(\frac{1+r_f}{1+(1-\tau)r_f}\right)^2\right) + \\ + \tau r_f \left(\frac{\mathrm{E}\left[\widetilde{FCF}_2^{\mathrm{u}}\right]}{\left(1+k^{E,\mathrm{u}}\right)^2} - \frac{(1-\tau)Div_2}{(1+(1-\tau)r_f)^2} \right) \left(1 + \frac{1+r_f}{1+(1-\tau)r_f}\right)$$

and from that

$$V_0^{\mathrm{l}} = 249.692 + 0.1 \times 0.5 \times \left(\frac{100}{1+0.15} - \frac{(1-0.5)\times 40}{1+(1-0.5)\times 0.1} \right) \times \left(1 + \left(\frac{1+0.1}{1+(1-0.5)\times 0.1}\right)^2\right) + \\ + 0.1 \times 0.5 \times \left(\frac{110}{(1+0.15)^2} - \frac{(1-0.5)\times 40}{(1+(1-0.5)\times 0.1)^2} \right) \times \left(1 + \frac{1+0.1}{1+(1-0.5)\times 0.1}\right) V_0^{\mathrm{l}} \approx 263.472$$

for the levered firm.

Managers now retain a fixed proportion of levered firm value. The more valuable the firm, the more managers retain. We assume that the factor

$$\widetilde{\mathrm{l}}_t = \frac{\widetilde{A}_t}{\widetilde{V}_t^{\mathrm{l}}}$$

is deterministic.

'Growth financed by constant intensity'

Valuation: *Valuation equation reads*

$$\widetilde{V}_t^{\mathrm{l}} = \sum_{s=t+1}^{T} \frac{\mathsf{E}\left[\prod_{h=t+1}^{s-1}\left(1-\left(1-\tau^D\right)l_h\right)\widetilde{FCF}_s^{\mathrm{u}}|\mathcal{F}_t\right]}{\prod_{h=t}^{s-1}(1+k_h)},$$

whereby

$$1+k_h = (1+k^{E,\mathrm{u}})\left(1-\frac{(1+r_f)\left(1-\tau^D\right)}{1+r_f\left(1-\tau^I\right)}l_h\right).$$

This equation is very similar to the Miles-Ezzell formula.

We assume a retention–value ratio of

$$\mathrm{l}_0 = \mathrm{l}_1 = \mathrm{l}_2 = 10\%\,.$$

The following then results for the cost of equity:

$$k_t = (1+k_t^{E,\mathrm{u}})\left(1-\frac{(1+r_f)(1-\tau)}{1+r_f(1-\tau)}\mathrm{l}_h\right)-1$$
$$= (1+0.15)\cdot\left(1-\frac{(1+0.1)\times(1-0.5)}{1+0.1\times(1-0.5)}\times 0.1\right)-1 \approx 8.976\%\,.$$

With that we get the value of the levered firm as

$$V_0^{\mathrm{l}} = \frac{\mathrm{E}\left[\widetilde{FCF}_1^{\mathrm{u}}\right]}{1+k} + \frac{(1-(1-\tau)l_1)\,\mathrm{E}\left[\widetilde{FCF}_2^{\mathrm{u}}\right]}{(1+k)^2} + \frac{(1-(1-\tau)l_1)(1-(1-\tau)l_2)\,\mathrm{E}\left[\widetilde{FCF}_3^{\mathrm{u}}\right]}{(1+k)^3}$$
$$\approx \frac{100}{1+0.08976} + \frac{(1-(1-0.5)\times 0.1)110}{(1+0.08976)^2} + \frac{(1-(1-0.5)\times 0.1)^2\times 121}{(1+0.08976)^3} \approx 264.137\,.$$

We considered four different retention policies and their impact on the firm value. These were

- autonomous retention,
- retention based on cash flows,
- retention based on dividends,
- retention based on market values.