Lecture: Autonomous Financing and Financing Based on Market Values I

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Discounted Cash Flow, Section 2.3, 2.4.1–2.4.3

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Outline

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Definition 2.4 (autonomous financing): *A firm is autonomously financed if its future amount of debt* $\widetilde{D}_t$ *is certain.*

Theorem 2.4 (APV): *If a firm is autonomously financed, then*

$$\widetilde{V}_t^{\mathrm{l}} = \widetilde{V}_t^{\mathrm{u}} + \sum_{s=t+1}^{T} \frac{\tau r_f D_{s-1}}{(1+r_f)^{s-t}}.$$

Proof: trivial.

Theorem 2.5 (constant debt, MoMi): *If $T = \infty$ and $D_t = const$, then*

$$\widetilde{V}_t^{\mathrm{l}} = \widetilde{V}_t^{\mathrm{u}} + \tau D_t.$$

Proof:

$$\begin{aligned}
\widetilde{V}_t^{\mathrm{l}} &= \widetilde{V}_t^{\mathrm{u}} + \sum_{s=t+1}^{T} \frac{\tau r_f D_{s-1}}{(1+r_f)^{s-t}} \\
&= \widetilde{V}_t^{\mathrm{u}} + \tau r_f D_t \sum_{s=t+1}^{T} \frac{1}{(1+r_f)^{s-t}} \\
&= \widetilde{V}_t^{\mathrm{u}} + \tau r_f D_t \frac{1}{r_f}.
\end{aligned}$$

An altered representation would be

$$\begin{aligned} V_0^{\mathrm{l}} &= V_0^{\mathrm{u}} + \tau D_0 \\ V_0^{\mathrm{l}} &= V_0^{\mathrm{u}} + \tau \mathrm{l}_0 V_0^{\mathrm{l}} \\ (1 - \tau \mathrm{l}_0) V_0^{\mathrm{l}} &= V_0^{\mathrm{u}} \\ V_0^{\mathrm{l}} &= \frac{\mathrm{E}\left[\widetilde{FCF}^{\mathrm{u}}\right]}{(1 - \tau \mathrm{l}_0) k^{E,\mathrm{u}}}. \end{aligned}$$

We come back to this (or at least a similar) equation in the next lecture when talking about the **Modigliani-Miller–adjustment**.

We now look at our finite example with $\tau = 50\%$.

$$D_0 = 100, \quad D_1 = 100, \quad D_2 = 50.$$

Hence,

$$\begin{aligned} V_0^{\mathrm{l}} &= V_0^{\mathrm{u}} + \frac{\tau r_f D_0}{1+r_f} + \frac{\tau r_f D_1}{(1+r_f)^2} + \frac{\tau r_f D_2}{(1+r_f)^3} \\ &\approx 229.75 + \frac{0.5 \times 0.1 \times 100}{1+0.1} + \frac{0.5 \times 0.1 \times 100}{(1+0.1)^2} \\ &\quad + \frac{0.5 \times 0.1 \times 50}{(1+0.1)^3} \approx 240.30. \end{aligned}$$

This is also the value of the firm threatened by default.

For later use we evaluate the future firm value $\widetilde{V}_1^{\mathrm{l}}$,

$$\begin{aligned}\widetilde{V}_1^{\mathrm{l}} &= \widetilde{V}_1^{\mathrm{u}} + \frac{\tau r_f D_1}{1+r_f} + \frac{\tau r_f D_2}{(1+r_f)^2} \\ &= \widetilde{V}_1^{\mathrm{u}} + \frac{0.5 \times 0.1 \times 100}{1+0.1} + \frac{0.5 \times 0.1 \times 50}{(1+0.1)^2} \\ &\approx \begin{cases} 199.88 & \text{if up,} \\ 164.74 & \text{if down.} \end{cases}\end{aligned}$$

Attention: The corresponding debt ratio

$$\widetilde{\mathrm{l}}_1 = \frac{D_1}{\widetilde{V}_1^{\mathrm{l}}} \approx \begin{cases} 50.03\ \% & \text{if up,} \\ 60.03\ \% & \text{if down,} \end{cases}$$

is **uncertain**!

Hence, a certain amount of debt implies an uncertain leverage ratio! (And vice versa. . .)

Since we can still use the APV-formula,

$$\widetilde{V}_0^l \approx 240.30.$$

Another way of obtaining this value is by evaluating $\mathrm{E}_Q[\widetilde{FCF}_t^l]$ and discounting it with the riskless rate.

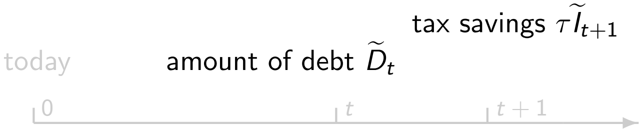
Here $\tau = 50\%$ and constant debt

$$D_t = 100.$$

Then

$$\begin{aligned} V_0^{\mathrm{l}} &= V_0^{\mathrm{u}} + \sum_{t=0}^{\infty} \frac{\tau r_f D_t}{(1+r_f)^{t+1}} \\ &= V_0^{\mathrm{u}} + \sum_{t=0}^{\infty} \frac{\tau r_f D_0}{(1+r_f)^{t+1}} \\ &= V_0^{\mathrm{u}} + \tau D_0 = 550. \end{aligned}$$

Definition 2.5 (financing based on market values): *Financing is based on market values if debt ratios l_t are certain.*



The time structure of financing based on market values

$\Longrightarrow$ The amount of future debt $\widetilde{D}_t$ is **uncertain**!

$\Longrightarrow$ The tax advantages from debt are **uncertain as well**!

$\Longrightarrow$ APV does not apply! Instead three different procedures. . .

To evaluate the company

1. We start with 'appropriate' cost of capital.
2. We assume that these cost of capital are deterministic and apply (as usual) a corresponding valuation formula.
3. We then look at the connection of these procedures: they are given by **textbook formulas**.

There are three 'appropriate' costs of capital, hence there will be **three valuation procedures**: FTE, TCF, WACC.

Notice that **default is not ruled out**!

Overview of three procedures:

procedure	**reference value**	**cost of capital**
FTE	$\widetilde{E}+\widetilde{D}$	$k^{E,l}$
TCF	$\widetilde{V}$	$\widetilde{k}^{\emptyset}$
WACC	$\widetilde{V}$	$\widetilde{WACC}$

These cost of capital are ratios of corresponding cash flows to the reference values. But there will be an anomaly with WACC...

With FTE we are looking at the stockholders and their *cost of equity*. The cash flow to stockholders is given by

	free cash flows	$\widetilde{FCF}^{\mathrm{l}}_{t+1}$
$-$	interest and repayment	$-\widetilde{I}_{t+1} - \widetilde{R}_{t+1}$.

Definition 2.6 (cost of equity): *Costs of equity are conditional expected returns*

$$\widetilde{k}_t^{E,\mathrm{l}} := \frac{\mathrm{E}\left[\widetilde{E}_{t+1} + \widetilde{FCF}^{\mathrm{l}}_{t+1} - \widetilde{I}_{t+1} - \widetilde{R}_{t+1} | \mathcal{F}_t\right]}{\widetilde{E}_t} - 1 \,.$$

Theorem 2.6 (FTE): *If $\widetilde{k}_t^{E,l}$ are deterministic, then*

$$\widetilde{E}_t = \sum_{s=t+1}^{T} \frac{\mathrm{E}\left[\widetilde{FCF}_s^{\mathrm{l}} - \widetilde{I}_s - \widetilde{R}_s | \mathcal{F}_t\right]}{\left(1 + k_t^{E,l}\right) \cdots \left(1 + k_{s-1}^{E,l}\right)}.$$

Proof: see our general valuation theorem (Theorem 1.1).
Remarks:

- FTE requires deterministic cost of equity.
- The theorem does not yet require financing based on market values!
- The leverage ratio does not appear in FTE.
- The knowledge of expected repayment is necessary.

Now we are looking at the stockholders and the debtholders, or the cost of equity and debt.

Definition 2.7 (weighted average cost of capital – type 1):
WACC type 1 are conditional expected returns

$$\widetilde{k}_t^{\emptyset} := \frac{\mathsf{E}\left[\widetilde{V}_{t+1}^{\mathrm{l}} + \widetilde{FCF}_{t+1}^{\mathrm{l}} | \mathcal{F}_t\right]}{\widetilde{V}_t^{\mathrm{l}}} - 1 \,.$$

Theorem 2.7 (TCF): *If $\widetilde{k}_t^{\emptyset}$ deterministic, then*

$$\widetilde{V}_t^{\mathrm{l}} = \sum_{s=t+1}^{T} \frac{\mathrm{E}\left[\widetilde{FCF}_s^{\mathrm{l}} | \mathcal{F}_t\right]}{\left(1 + k_t^{\emptyset}\right) \cdots \left(1 + k_{s-1}^{\emptyset}\right)} .$$

Proof: see our general valuation theorem (Theorem 1.1).

Remarks:

- TCF requires deterministic WACC type 1.
- The theorem does not yet require financing based on market values!
- The leverage ratio does not appear in TCF.
- The knowledge of expected debt is not necessary.

What is the connection between FTE and TCF? The answer is the textbook formula.

Theorem 2.8 (TCF textbook formula): *It always holds that*

$$\widetilde{k}_t^{\emptyset} = \widetilde{k}_t^{E,\mathrm{l}} \left(1 - \widetilde{\mathrm{l}}_t\right) + \widetilde{k}_t^{D} \widetilde{\mathrm{l}}_t \ .$$

$$
\begin{aligned}
\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t &= \mathrm{E}\left[\widetilde{E}_{t+1}+\widetilde{FCF}^{\mathrm{l}}_{t+1}-\widetilde{R}_{t+1}+\widetilde{D}_{t+1}-\widetilde{D}_{t+1}-\widetilde{I}_{t+1}|\mathcal{F}_t\right]\\
\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t &= \mathrm{E}\left[\widetilde{V}^{\mathrm{l}}_{t+1}+\widetilde{FCF}^{\mathrm{l}}_{t+1}-\widetilde{R}_{t+1}-\widetilde{D}_{t+1}-\widetilde{I}_{t+1}|\mathcal{F}_t\right]\\
\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t &= \mathrm{E}\left[\widetilde{V}^{\mathrm{l}}_{t+1}+\widetilde{FCF}^{\mathrm{l}}_{t+1}-\widetilde{D}_t-\widetilde{k}_t^D\,\widetilde{D}_t|\mathcal{F}_t\right]\\
\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t+\left(1+\widetilde{k}_t^D\right)\widetilde{D}_t &= \mathrm{E}\left[\widetilde{V}^{\mathrm{l}}_{t+1}+\widetilde{FCF}^{\mathrm{l}}_{t+1}|\mathcal{F}_t\right]\\
\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t+\left(1+\widetilde{k}_t^D\right)\widetilde{D}_t &= \left(1+\widetilde{k}_t^{\emptyset}\right)\widetilde{V}^{\mathrm{l}}_t\\
\widetilde{E}_t+\widetilde{k}_t^{E,\mathrm{l}}\widetilde{E}_t+\widetilde{D}_t+\widetilde{k}_t^D\widetilde{D}_t &= \widetilde{V}^{\mathrm{l}}_t+\widetilde{k}_t^{\emptyset}\,\widetilde{V}^{\mathrm{l}}_t\\
\widetilde{k}_t^{E,\mathrm{l}}\,\frac{\widetilde{E}_t}{\widetilde{V}^{\mathrm{l}}_t}+\widetilde{k}_t^D\,\frac{\widetilde{D}_t}{\widetilde{V}^{\mathrm{l}}_t} &= \widetilde{k}_t^{\emptyset}.
\end{aligned}
$$

The costs of debt are not reduced by the tax rate in the TCF textbook formula. The formula holds regardless of whether the relevant variables are deterministic or stochastic.

In particular: financing based on market values is not necessary!

Assume no default. One of two cases possible

market-value financing If WACC type 1 or cost of equity are deterministic, the other is deterministic as well. TCF and FTE only used simultaneously.

non market-value financing Either WACC type 1 or cost of equity has to be uncertain. TCF and FTE exclude each other.

Proof:

$$\widetilde{k}_t^{\emptyset} = \widetilde{k}_t^{E,l} \left(1 - \widetilde{l}_t\right) + r_f \widetilde{l}_t \ .$$

We are now at stockholders and debtholders again.

Definition 2.8 (weighted average cost of capital – type 2):
WACC type 2 are the conditional expected returns

$$\widetilde{WACC}_t := \frac{\mathsf{E}\left[\widetilde{V}^{\mathrm{l}}_{t+1} + \widetilde{FCF}^{\mathrm{u}}_{t+1} | \mathcal{F}_t\right]}{\widetilde{V}^{\mathrm{l}}_t} - 1 \,.$$

Remark: These are costs of capital of a firm that is on the one hand levered ($\widetilde{V}^{\mathrm{l}}_t$) and on the other hand unlevered ($\widetilde{FCF}^{\mathrm{u}}_{t+1}$).
Apples and oranges mixed here.

Theorem 2.9 (WACC): *If $\widetilde{WACC}_t$ is deterministic, then*

$$\widetilde{V}_t^{\mathrm{l}} = \sum_{s=t+1}^{T} \frac{\mathrm{E}\left[\widetilde{FCF}_s^{\mathrm{u}} | \mathcal{F}_t\right]}{(1 + WACC_t) \cdots (1 + WACC_{s-1})}.$$

Proof: see our general valuation theorem (Theorem 1.1)

Remarks:

- WACC requires deterministic WACC type 2.
- The theorem above does not yet require financing based on market values!
- The leverage ratio does not appear in WACC.
- The knowledge of cash flow of an unlevered firm is necessary.

What is the connection between FTE and WACC? The answer is another textbook formula.

Theorem 2.10 (WACC textbook formula): *Always*

$$\widetilde{WACC}_t = \widetilde{k}_t^{E,\mathrm{l}} \left(1 - \widetilde{\mathrm{l}}_t\right) + \widetilde{k}_t^{D} \left(1 - \tau\right) \widetilde{\mathrm{l}}_t.$$

Proof

$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t = \mathsf{E}\left[\widetilde{E}_{t+1} + \widetilde{FCF}^{\mathrm{l}}_{t+1} - \widetilde{R}_{t+1} - \widetilde{I}_{t+1}|\mathcal{F}_t\right]$$

$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t = \mathsf{E}\left[\widetilde{V}^{\mathrm{l}}_{t+1} + \widetilde{FCF}^{\mathrm{l}}_{t+1} - \widetilde{R}_{t+1} - \widetilde{I}_{t+1} - \widetilde{D}_{t+1}|\mathcal{F}_t\right]$$

$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t = \mathsf{E}\left[\widetilde{V}^{\mathrm{l}}_{t+1} + \widetilde{FCF}^{\mathrm{u}}_{t+1} - \widetilde{R}_{t+1} - \widetilde{I}_{t+1} - \widetilde{D}_{t+1} + \tau(\widetilde{I}_{t+1} - \widetilde{D}_t + \widetilde{R}_{t+1} + \widetilde{D}_{t+1})|\mathcal{F}_t\right]$$

$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t = \mathsf{E}\left[\widetilde{V}^{\mathrm{l}}_{t+1} + \widetilde{FCF}^{\mathrm{u}}_{t+1} - (1+\widetilde{k}^D)\widetilde{D}_t + \tau\,\widetilde{k}^D\widetilde{D}_t|\mathcal{F}_t\right].$$

$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t+\left(1+\widetilde{k}^D(1-\tau)\right)\widetilde{D}_t=\mathsf{E}\left[\widetilde{V}_{t+1}^{\mathrm{l}}+\widetilde{FCF}_{t+1}^{\mathrm{u}}|\mathcal{F}_t\right]$$

$$\left(1+\widetilde{k}_t^{E,\mathrm{l}}\right)\widetilde{E}_t+\left(1+\widetilde{k}^D(1-\tau)\right)\widetilde{D}_t=(1+WACC_t)\,\widetilde{V}_t^{\mathrm{l}}$$

$$\widetilde{E}_t+\widetilde{k}_t^{E,\mathrm{l}}\widetilde{E}_t+\widetilde{D}_t+\widetilde{k}^D(1-\tau)\widetilde{D}_t=\widetilde{V}_t^{\mathrm{l}}+WACC_t\,\widetilde{V}_t^{\mathrm{l}}$$

$$\widetilde{k}_t^{E,\mathrm{l}}\,\frac{\widetilde{E}_t}{\widetilde{V}_t^{\mathrm{l}}}+\widetilde{k}^D(1-\tau)\,\frac{\widetilde{D}_t}{\widetilde{V}_t^{\mathrm{l}}}=WACC_t.$$

And this was to be shown QED

The costs of debt are reduced by the tax rate in the WACC textbook formula. The formula holds regardless of whether the relevant variables are deterministic or stochastic.

In particular: financing based on market values is not necessary!

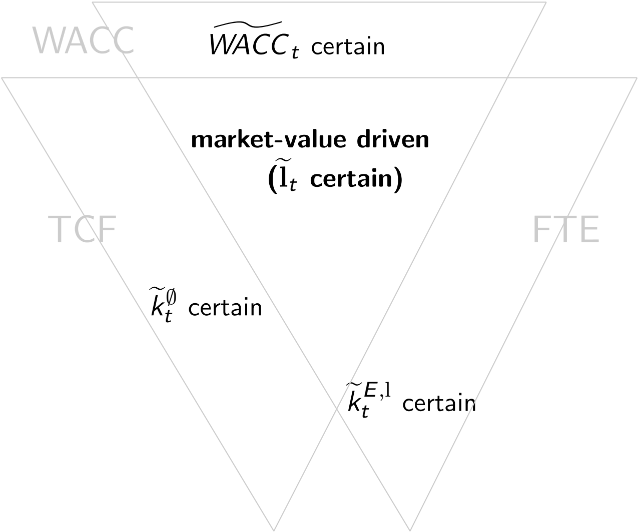
Assume no default. One of two cases possible

market-value financing If WACC type 2 or cost of equity are deterministic, the other is deterministic as well. WACC and FTE only used simultaneously.

non market-value financing Either WACC type 2 or cost of equity has to be uncertain. WACC and FTE exclude each other.

Proof:

$$\widetilde{WACC}_t = \tilde{k}_t^{E,l} \left(1 - \tilde{l}_t\right) + r_f \left(1 - \tau\right) \tilde{l}_t.$$


WACC
$\widetilde{WACC}_t$ certain
market-value driven
($\widetilde{l}_t$ certain)
TCF
FTE
$\widetilde{k}_t^{\emptyset}$ certain
$\widetilde{k}_t^{E,l}$ certain