Lecture: Levered Firms

Lutz Kruschwitz & Andreas Löffler

Discounted Cash Flow, Section 3.2

Remark: The slightly expanded second edition (Springer, open access) has different enumeration than the first (Wiley). We use Springer's enumeration in the slides and Wiley's in the videos.

,

Outline

,

Let us have a closer look at levered firms: their value $\widetilde{V}_t^{\mathrm{l}}$ equals debt ($\widetilde{D}_t$) plus equity ($\widetilde{E}_t$)

$$\widetilde{V}_t^{\mathrm{l}} = \widetilde{D}_t + \widetilde{E}_t.$$

These are **market values**, not book values! Debt is granted at time t, after one period the debtor has to pay redemption and interest $\widetilde{I}_{t+1}$. **Without default**

$$\widetilde{I}_{t+1} = r_f \widetilde{D}_t.$$

(Default will be discussed later.)

Debt ratio and leverage ratio are two important numbers. Both can be uncertain:

$$\left.\begin{array}{lr} \text{debt ratio} & \widetilde{\mathrm{l}}_t = \frac{\widetilde{D}_t}{\widetilde{V}_t^{\mathrm{l}}} \\[2em] \text{leverage ratio} & \widetilde{L}_t = \frac{\widetilde{D}_t}{\widetilde{E}_t} \end{array}\right\} \text{implies } \widetilde{L}_t = \frac{\widetilde{\mathrm{l}}_t}{1-\widetilde{\mathrm{l}}_t}.$$

We now introduce the book value of a company. These are those values, with which the owners' or creditors' claims are to be **found in the balance sheets**.

The book value $\underline{\widetilde{V}}^{\mathrm{l}}_t$ equals book value of debt ($\underline{\widetilde{D}}_t$) plus book value of equity ($\underline{\widetilde{E}}_t$)

$$\underline{\widetilde{V}}^{\mathrm{l}}_t = \underline{\widetilde{D}}_t + \underline{\widetilde{E}}_t.$$

Again debt ratio and leverage ratio can be defined

$$\underline{\widetilde{\mathrm{l}}}_t = \frac{\underline{\widetilde{D}}_t}{\underline{\widetilde{V}}^{\mathrm{l}}_t} \qquad \text{and} \qquad \underline{\widetilde{L}}_t = \frac{\underline{\widetilde{D}}_t}{\underline{\widetilde{E}}_t}.$$

$=$	Earnings before taxes	$\widetilde{EBT}$
$+$	Interest	$r_f\,\widetilde{D}$
$=$	Earnings before interest and taxes	$\widetilde{EBIT}$
$+$	Accruals	$\widetilde{Accr}$
$=$	Gross cash flow before taxes	$\widetilde{GCF}$
$-$	Taxes	$\widetilde{Tax}$
$-$	Investment expenses	$\widetilde{Inv}$
$=$	Free cash flow	$\widetilde{FCF}$

Only the red items differ for levered and unlevered firms.

Assumption 3.2 (identical gross cash flows): *There is no difference between levered and unlevered firms with respect to gross cash flows, accruals, investments.*

Earnings before taxes are the tax base. Hence we have for the tax payments of the corporation

$$\widetilde{Tax} = \tau \widetilde{EBT}. \qquad (3.8)$$

Because $\widetilde{EBIT}^{\mathrm{u}} = \widetilde{EBIT}^{\mathrm{l}}$, the tax payments of the levered and the unlevered firm differ by

$$\widetilde{Tax}_t^{\mathrm{u}} - \widetilde{Tax}_t^{\mathrm{l}} = \tau r_f \widetilde{D}_{t-1}.$$

This difference is called the **tax shield** from debt.

Interest is determined by the riskless rate r_f and debt $\widetilde{D}_{t-1}$. The tax shield is

$$\tau r_f \widetilde{D}_{t-1}.$$

From our assumptions **only debt can be uncertain in this model!**
$\Longrightarrow$ The level of tax shield is determined by the **financing policy**.

The aim of DCF is the **valuation of these (uncertain) tax advantages**. Not less, but also not more.

1. **Autonomous financing**: future amount of debt $\widetilde{D}_t$ is fixed.
2. **Financing based on market values**: evaluator sets the future debt ratios based on market values $\widetilde{\mathrm{l}}_t$.
3. **Financing based on book values**: the future debt ratios to book values $\widetilde{\underline{\mathrm{l}}}_t$ are fixed.
4. **Financing based on cash flows**: amount of debt is based on the firm's cash flows.
5. **Financing based on dividends**: debt managed so that previously determined dividend distributed.
6. **Financing based on dynamical leverage ratio**: evaluator sets the future cash flow–debt ratios.

We do not want to answer the question as to which of these financing policies is particularly close to reality. Further, we will not discuss the question of which of the mentioned financing policies maximizes the value of the levered company (later we will see: extended leverage increases company value).

Assumption 3.3 (given debt policy): *The debt policy of the firm (although probably uncertain) is already prescribed.*

Our aim is a **general valuation equation** for the tax shield.

Free cash flows differ only by tax payments (see slide 4). Using the definition of tax base (3.8)

$$\widetilde{FCF}^{\mathrm{l}}_t = \widetilde{FCF}^{\mathrm{u}}_t + \tau r_f \widetilde{D}_{t-1} \qquad (3.9)$$

From the fundamental theorem (Theorem 2.2) for levered and unlevered firms we get

$$\widetilde{V}_t^{l,u} = \frac{\mathrm{E}_Q\left[\widetilde{FCF}^{\mathrm{l,u}}_{t+1} + \widetilde{V}^{\mathrm{l,u}}_{t+1} | \mathcal{F}_t\right]}{1+r_f}.$$

It follows that

$$\widetilde{V}_t^{\mathrm{l}} - \widetilde{V}_t^{\mathrm{u}} = \frac{\mathrm{E}_Q\left[\tau r_f \widetilde{D}_t + \widetilde{V}^{\mathrm{l}}_{t+1} - \widetilde{V}^{\mathrm{u}}_{t+1} | \mathcal{F}_t\right]}{1+r_f}.$$

Then

$$\widetilde{V}_t^{\mathrm{l}} = \widetilde{V}_t^{\mathrm{u}} + \frac{\mathsf{E}_Q\left[\tau r_f \widetilde{D}_t | \mathcal{F}_t\right]}{1+r_f} + \ldots + \frac{\mathsf{E}_Q\left[\tau r_f \widetilde{D}_{T-1} | \mathcal{F}_t\right]}{(1+r_f)^{T-t}}.$$

This gives finally

$$\widetilde{V}_t^{\mathrm{l}} = \widetilde{V}_t^{\mathrm{u}} + \frac{\tau r_f \,\mathsf{E}_Q\left[\widetilde{D}_t | \mathcal{F}_t\right]}{1+r_f} + \ldots + \frac{\tau r_f \,\mathsf{E}_Q\left[\widetilde{D}_{T-1} | \mathcal{F}_t\right]}{(1+r_f)^{T-t}}. \tag{3.11}$$

This is the **basic equation** for valuing the tax shield. It clearly shows a dependence of the tax shield on the financing policy (future debt levels).

Default trigger can be, for example:

- lack of liquidity,
- debt greater than assets,
- expected lack of liquidity in the near future.

At the moment we will **not specify the default trigger.**

If bankruptcy occurs, there are three possibilities:

- restructuring the company,
- liquidation of the company, or
- sell-out of the remaining assets.

We will only assume that all conceivable developments were taken into consideration when determining the company's cash flows.

Owners have no personal liability.

Creditors and shareholders have identical information about the company, its estate, the firm's cost of capital and its financing policy.

Assumption 3.5 (gross cash flows and default): *The gross cash flows as well as the investment and accruals policy of the unlevered firm do not differ from those of the firm in danger of default.*

Who gets what in case of default?

Assumption 3.6 (prioritization of debt): *The tax office's claims range before those of other creditors. The cash flows are always sufficient to at least pay off the tax debts in full.*

Now a new notation is necessary:

$\widetilde{D}_t$	amount of credit outstanding in t
$\widetilde{R}_{t+1}$	amount that is paid back in $t+1$
$\widetilde{I}_{t+1}$	interest paid in $t+1$

The tax office allows interest $\widetilde{I}_{t+1}$ to be deducted from the tax base. On the other hand, the cancellation of debt

$$\underbrace{\widetilde{D}_t - \widetilde{D}_{t+1}}_{\text{should be paid back}} \quad - \quad \underbrace{\widetilde{R}_{t+1}}_{\text{was paid back}}$$

adds to the tax base ('recapitalization gain'). Hence, the **tax due** for a firm in danger of default is

$$\widetilde{Tax}^{\mathrm{l}}_{t+1} = \tau\Big(\widetilde{GCF}_{t+1} - \widetilde{Accr}_{t+1} - \underbrace{\widetilde{I}_{t+1}}_{\substack{\text{interest,}\\ \text{deducted}}} + \underbrace{\widetilde{D}_t - \widetilde{D}_{t+1} - \widetilde{R}_{t+1}}_{\substack{\text{gain,}\\ \text{added}}}\Big).$$

Again from slide 4 we have

$$\widetilde{FCF}^{\mathrm{l}}_{t+1} = \widetilde{FCF}^{\mathrm{u}}_{t+1} + \widetilde{Tax}^{\mathrm{u}}_{t+1} - \widetilde{Tax}^{\mathrm{l}}_{t+1}$$
$$= \widetilde{FCF}^{\mathrm{u}}_{t+1} + \tau\left(\widetilde{I}_{t+1} + \widetilde{R}_{t+1} + \widetilde{D}_{t+1} - \widetilde{D}_{t}\right)$$

and the **main valuation equation** now reads

$$\widetilde{V}^{\mathrm{l}}_t = \widetilde{V}^{\mathrm{u}}_t + \sum_{s=t+1}^{T} \frac{\tau\, \mathrm{E}_Q\left[\widetilde{I}_s + \widetilde{R}_s + \widetilde{D}_s - \widetilde{D}_{s-1} | \mathcal{F}_t\right]}{(1+r_f)^{s-t}}. \qquad (3.15)$$

This does not look like (3.11)?! But let us see…

The **debtholders behave rational**. Therefore, at time s

$$\widetilde{D}_{s-1} = \frac{\mathrm{E}_Q\left[\text{value in } s|\mathcal{F}_{s-1}\right]}{1+r_f} = \frac{\mathrm{E}_Q\left[\widetilde{R}_s + \widetilde{D}_s + \widetilde{I}_s|\mathcal{F}_{s-1}\right]}{1+r_f}$$

and from rule 2 and rule 5 for all $t \leq s$

$$r_f\,\mathrm{E}_Q\left[\widetilde{D}_{s-1}|\mathcal{F}_t\right] = \mathrm{E}_Q\left[\widetilde{I}_s + \widetilde{R}_s + \widetilde{D}_s - \widetilde{D}_{s-1}|\mathcal{F}_t\right].$$

And the rhs is the nominator from the main valuation equation (3.15) above!

Even in the case of default the valuation equation (3.11)

$$\widetilde{V}_t^{\mathrm{l}} = \widetilde{V}_t^{\mathrm{u}} + \frac{\tau r_f \operatorname{E}_Q\left[\widetilde{D}_t|\mathcal{F}_t\right]}{1+r_f} + \ldots + \frac{\tau r_f \operatorname{E}_Q\left[\widetilde{D}_{T-1}|\mathcal{F}_t\right]}{(1+r_f)^{T-t}}$$

holds.

Or: default does not make the DCF theory fail. The difficulties of taking default into consideration lie much more in the fact that the relevant financing policies must be formulated with care.

Someone who invests $\widetilde{D}_t$ today is entitled to payments amounting to $\widetilde{D}_t + \widetilde{I}_{t+1}$ less remission of debts $\widetilde{D}_t - \widetilde{D}_{t+1} - \widetilde{R}_{t+1}$. Hence,

Definition 3.4 (cost of debt): *The cost of debt of a levered firm is*

$$\widetilde{k}_t^D = \frac{\mathrm{E}[\widetilde{D}_{t+1} + \widetilde{I}_{t+1} + \widetilde{R}_{t+1} | \mathcal{F}_t]}{\widetilde{D}_t} - 1.$$

If there is no default $\widetilde{k}_t^D = r_f$.

We do not require the cost of debt to be deterministic today. Cost of debt will not be used itself to determine the value of firms.

Provisional leverage policy may be

$$D_0 = 100, \qquad D_1 = 100, \qquad D_2 = 50.$$

Tax rate is $\tau = 50\%$.

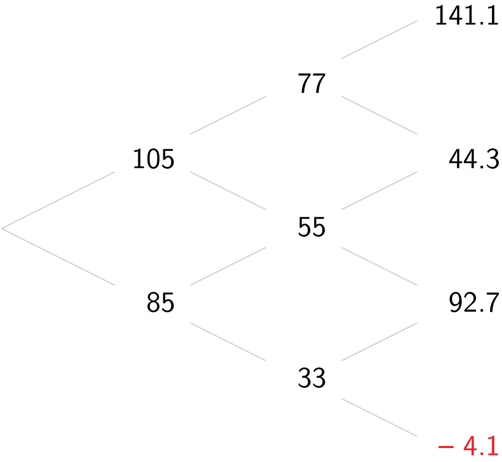
Bankruptcy enters in if in state ω,

$$\begin{cases} \widetilde{FCF}^{\mathrm{l}}_t(\omega) < \widetilde{I}_t(\omega) + D_{t-1} - D_t & \text{for a levered firm,} \\ \widetilde{FCF}^{\mathrm{u}}_t(\omega) < 0 & \text{for an unlevered firm.} \end{cases}$$

Here: bankruptcy is equal to lack of liquidity.

Shareholder's claims 20

Does bankruptcy occur? $\widetilde{FCF}^{\mathrm{u}}_t(\omega) + \tau r_f D_{t-1} - (1 + r_f) D_{t-1} + D_t$



In all states except *dd* (and following) the creditors demand

$$\omega \neq dd \quad \Longrightarrow \quad k_t^{\mathrm{D,nominal}}(\omega) = r_f\,.$$

Only in state $\omega = dd$ bankruptcy can occur. Hence,

$$k_t^{\mathrm{D,nominal}}(dd) > r_f\,.$$

We now want to determine $k_t^{\mathrm{D,nominal}}(dd)$.

Let us look at the states that follow dd. At state ddd the claims cannot be paid off in full. The cash flow is necessary to pay interest and pay back the loan. Therefore,

$$\widetilde{FCF}_3^{\mathrm{l}}(ddd) + \underbrace{D_3}_{=0} = \widetilde{R}_3(ddd) + \widetilde{I}_3(ddd).$$

On the other hand, since the tax office claims have priority,

$$\widetilde{FCF}_3^{\mathrm{l}}(ddd) = \widetilde{FCF}_3^{\mathrm{u}}(ddd) + \tau(\widetilde{I}_3(ddd) + \widetilde{R}_3(ddd) + \underbrace{D_3}_{=0} - D_2)$$

which finally gives

$$\widetilde{FCF}_3^{\mathrm{l}}(ddd) = \frac{1}{1-\tau}\left(\widetilde{FCF}_3^{\mathrm{u}}(ddd) - \tau D_2\right) = 46.8.$$

Determining $k_t^{\mathrm{D,nominal}}(dd)$ 23

At state *ddu* the creditors get

$$(1 + k_2^{D,nominal}(dd))D_2 = \widetilde{R}_3(ddu) + \widetilde{I}_3(ddu).$$

Using the fundamental theorem we must have

$$D_2 = \frac{\mathrm{E}_Q[\widetilde{R}_3 + \widetilde{I}_3|\mathcal{F}_2]}{1 + r_f}$$

which finally gives

$$k_2^{D,nominal}(dd) \approx 32.962\%$$

The resulting cost of debt are

$$\widetilde{k}_2^D(dd) = \frac{\left(1+\widetilde{k}_2^{D,nom}(dd)\right) D_2\, P_3(u|dd) + \frac{1}{1-\tau}\left(\widetilde{FCF}_3^{\mathrm{u}}(ddd) - \tau D_2\right) P_3(d|dd)}{D_2} - 1$$

$$\approx \frac{(1+0.32962)\times 50\times 0.5 + \frac{1}{1-0.5}(48.4-0.5\times 50)\times 0.5}{50} - 1 \approx 13.281\,\%.$$

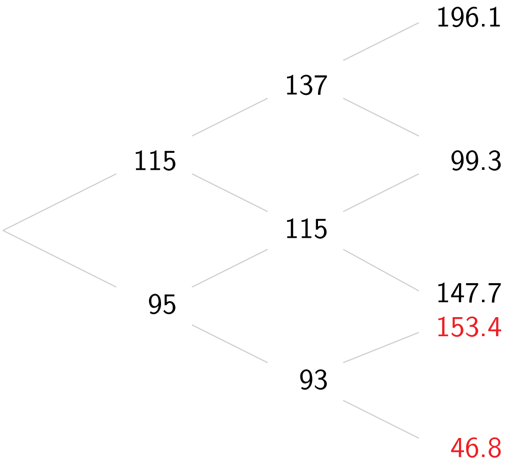
From these results we get

$$\begin{aligned}
\widetilde{FCF}^{\mathrm{l}}_3(ddu) &= \widetilde{FCF}^{\mathrm{u}}_3(ddu) + \tau(\widetilde{I}_3(ddu) + \widetilde{R}_3(ddu) + \widetilde{D}_3 - \widetilde{D}_2) \\
&= 145.2 + 0.5 \cdot 0.32962 \cdot 50 \\
&\approx 153.44\,.
\end{aligned}$$

This is enough to pay the (nominal) interest rate of almost 33% in *ddu*.

Notice, that the binomial tree is now not recombining.

Cash flows $\widetilde{FCF}^1$ with default risk



We consider levered and unlevered firms with identical gross cash flows.

The tax advantage for the levered firm (no default) is $\tau r_f \widetilde{D}_{t-1}$.

The financing policy of the firm determines the value of the tax shield.

DCF remains valid even in the case of default. But then the relevant financing policy must be handled with care.